Wassily Leontief's The Structure of the American Economy

Dr. Fidel Aroche

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Abstract: In 1941 Wassily W. Leontief published his seminal book The Structure of the American Economy 1919-1929; ten years later, in 1951, a second edition appeared as The Structure of the American Economy 1919-1939, containing four further chapters that first had appeared as journal articles, three of them between 1944 and 1946 and the fourth one in 1949. The author states that this book contains an applied study of interdependence between sectors in an economy within the general equilibrium framework (interdependence and general equilibrium are equivalent). The Tableau Économique by François Quesnay is taken as example and precedent. Interestingly, the issues discussed in Leontief's book can easily be linked to the series or papers the author had published during the 1930's. This paper revises some of the salient features of Leontief's The Structure of the American Economy in relation to the –then-current discussions, as well as the reactions the book generated by the time it appeared. In fact there are a number of comments published in various journals during the 1940's and 1950's that make it evident the importance its contemporaries gave to the book**.**

Wassily Leontief's *The Structure of the American Economy*

In 1941 Wassily W. Leontief, professor of Economics in Harvard University published a seminal book *The Structure of the American Economy 1919-1929*, containing three parts and six chapters. In this book, the author explains the principles of a model that would later be known as Input-Output (IO) in its "closed" version and the process of construction of the database necessary to operate the model. That database is based on the US census of 1919 and 1929. Ten years later, in 1951, a second edition of the book appeared as *The Structure of the American Economy 1919-1939*, containing an additional Part IV and four further chapters that had appeared first as journal articles; three of them between 1944 and 1946 in the *Quarterly Journal of Economics* and the fourth one in 1949 in the *American Economic Review*. These further chapters explain the "open" version of the IO model. The book includes a, IO table of the US economy

for 1939, prepared by the Bureau of Labor (Sic.) Statistics of the US government.
Leontief states that this book contains an applied study of interdependence between sectors in an economy within the general equilibrium framework (interdependence and general equilibrium are used as equivalent). The *Tableau Économique* by François Quesnay is taken as example and precedent. Interestingly, the issues discussed in Leontief's book can easily be linked to the series or papers the author published during the 1930's. This paper revises some of the salient features of Leontief's *The Structure of the American Economy* in relation to the –then current discussions- as well as the reactions the book generated by the time it appeared. In fact there are a number of comments published in various journals during the 1950's that make it evident the importance its contemporaries gave to the book**.**

- The introduction explains the idea of the economy as a system, where all its parts are interconnected and then, Leontief describes the plan of the book, explaining some of the main ideas that led him in the process of writing. One of the main features that characterise Leontief is his preoccupation about the usefulness of his work for empirical research. This section ends warning the reader that the conclusions and the findings of this book are not easy to summarise, because they are concrete and particular for the economy for which the Tableau was built; besides in a footnote it is stated that Parts II and III contain mathematical analysis, which difficult reading the piece of work. *Mathematical Analysis for Economists* by R.D.G. Allen (Macmillan, 1939) is the advised bibliography to get the skills necessary to surpass these obstacles.

- Part I is called "Quantitative Input and Output Relations in the Economic System of the United States in 1919 and 1929" where the first remark is that the statistical study presented is an attempt to construct a *Tableau Economic* for the United States using the available statistics for 1919 and 1929, mainly published by the government. In 1919 the economy grows vigorously and in 1929 the cycle turns towards recession. Then a few basic concepts on revenue and costs accounts.

The purpose of the model is to cover the whole economic activity of the country, including the consumers; the intention being to construct a closed model. In principle there are two accounts for each firm and each household, corresponding to a period of time: expenditure and revenue. These accounts cover every trade that each individual performs during that period of time.
Since an individual's expenditure corresponds to an income of another one, these accounts can be arranged in a matrix that can be read as usual, on the columns the outlays (input purchases) and on the rows the revenues. Individual accounts can be grouped by industries, by regions or by any other principle. Presumably these two criteria can also be combined and have a regional and industrial matrix. Reducing the number of accounts can also continue to the point of having a single box.
By adding every household in one account and every firm in another, the matrix is reduced to four boxes. The sum of total household expenditures is equal to the total value of services credited to households and that equals the national income, since the

households are the owners of the productive factors. Hence the value added in the productive activities is equal to the national income and that equals also the value of the goods supplied by the businesses to the households. These identities hold in a static economy.
Part I explains also the statistical procedure to understand the IO tables included with the book. These can be analysed from three viewpoints: from the distribution of output, from the production costs and from the industrial balance between costs and revenues. It is interesting to see that the empirical problems Leontief faced when building the IO table, remain the same, e.g. those derived classifying transport margins, taxes or the valuation of imports.
The remuneration of labour services presents no difficulty to identify as the sum of wages and salaries disbursed by each firm. The payment to the capital factor, on the contrary are not easy to define, as that contains the retribution to the capital services, as well as entrepreneurial returns, monopolistic revenues, windfall profits, interests, dividends, undistributed profits. The difficulty with this variable lies on the distance between the theoretical concept of capital as a factor, the measurement of its productivity and the actual measurement of the revenues that the accountants assign to this variable.

- Part II is titled "The Theoretical Scheme" and indeed it begins by stating that the model contained in this book is based on a general equilibrium system, which allows analysing the complexities of an interdependent economy, from a static perspective. Leontief regrets that the dynamic general equilibrium model remained at that time an unwritten chapter of economic theory. The reasons for choosing such a theoretical framework is that it enables to take into account the network of interrelationships between the different sectors in the system. Leontief replays to the critics of the general equilibrium perspective that despite its failures, and that it cannot cope with the actual economic processes, it allows a deeper understanding of the economy if compared to the partial approach.

The general equilibrium system suggested here, takes the relative quantities and prices as unknown variables, whereas the technical conditions of production are taken as data. This system had been presented in the paper of 1937 "Interrelation of Prices, Output, Savings and Investment. A Study in Empirical Application of the Economic Theory of General Interdependence" published in *The Review of Economics and Statistics*. Three sets of equations represent the model of general interdependence presented by Leontief. An assumption at this point is that the system is in stationary equilibrium: simple reproduction with no savings, investments or changing capital stocks. Linear equations I describe the fact that total output (quantities) in each industry equals the sum of its products consumed by other industries:

(I)

$$-X_1 + x_{21} + x_{31} + \ldots + x_{i1} + \ldots + x_{n1} = 0$$
$$x_{12} - X_2 + x_{32} + \ldots + x_{i2} + \ldots + x_{n1} = 0$$
$$x_{13} + x_{23} - X_3 + \ldots + x_{i1} + \ldots + x_{n1} = 0$$

$$\ldots$$
$$x_{1n} + x_{2n} + x_{3n} + \ldots + x_{in} + \ldots - X_n = 0$$

Equations II state that the former is also true in value terms (price times quantities)

(II)

$$-X_1P_1 + x_{12}P_2 + x_{13}P_3 + \ldots + x_{1i}P_i + \ldots + x_{1n}P_n = 0$$
$$x_{21}P_1 + X_2P_2 + x_{23}P_3 + \ldots + x_{2i}P_i + \ldots + x_{2n}P_n = 0$$
$$x_{31}P_1 + x_{32}P_2 - X_3P_3 + \ldots + x_{3i}P_i + \ldots + x_{3n}P_n = 0$$
$$\ldots$$
$$x_{n1}P_1 + x_{n2}P_2 + x_{n3}P_3 + \ldots + x_{ni}P_i + \ldots - X_nP_n = 0$$

where P_i are the prices of the products produced in the *n* industries. Equations III describe the production functions in each industry and cannot be derived on purely assumptions, but ought to be related to the real technical conditions of production processes. Nevertheless at the time there were not enough empirical studies for having empirical data for each economic activity. Thus Leontief chooses the most rigid type of production functions, "… the amount of each cost element is assumed to be strictly proportioned to the quantity of output … we describe the technical setup of each industry by a series of as many homogeneous linear equations as there are separate cost factors involved:" (p. 37).

$$x_{i1} = a_{i1}X_i,\ x_{i2} = a_{i2}X_i,\ \ldots,\ x_{in} = a_{in}X_i$$

Following Walras' terminology, the constants a_{i1}, a_{i2}, …, a_{in}, … are referred as the coefficients of production. Leontief explains both in this book and in his 1937 paper that the choice of such production functions allows him to reject the marginal productivity theory, excluding technical substitutability of factors in any production process. Factors are complementary: and increasing the amount employed of any of them results in no increase of the output, unless every input is increased proportionately. Leontief argues extensively about the implications of this assumption. Basically one can also argue with him that given the lack of practical knowledge about the actual production functions of the various productive activities that he refers to, such an assumption is as valid as any other one. Moreover, as I have also written elsewhere (Aroche, 2006), it is also possible to say that at any one time the researcher faces just one point of the isoquant, which is also expected to be of equilibrium, and one can also predict the shape of that isoquant, but there is no certainty about the path that technology will follow.

Leontief had also investigated about the shape of the demand and supply curves, which published as "Ein Versuch zur statischen Analyse von Angebot und Nachfrage" (An Attempt for a Statistical Analysis of Supply and Demand), in *Welwirtschaftliches Archiv* in 1929 and his research had led him to the well known controversy with Ragnar Frisch (Aroche, 2008) on the statistical evidence and analysis of supply and demand functions, as well as on index numbers and the problems with aggregation.

As it has been stated earlier in this paper, the 1941 edition of *The* Structure *of the American Economy* presents a closed model, thus, the households are treated as any other "industry." First of all, Leontief states that the consumer's and the producer's theories are formally not very different, for example, the isoquants and the indifference curves are analogous; second, the service output of a household can be assimilated to the idea of a firm's output. Third, this model is not related to the cost theories of wages and the consumer's expenditures are linked to his earnings. Further, consumers seem to keep stable consuming patterns and the degree of substitutability among expenditures seem to vary with the commodity classification chosen and the one taken in the particular study increases the non substitutability.

The second set of equations can be rewritten in terms of prices as dependent variables and the technical coefficients as independent ones. The system will be homogeneous of degree zero because (as a general equilibrium system) it determines relative prices only, that is, if all prices (the price vector) are multiplied by some number (scalar), every real individual supply and demand for any commodity remains unchanged. These arguments had already been presented by Leontief in his 1936 paper "The Fundamental Assumption of Mr Keynes' Monetary Theory of Unemployment" in the *Quarterly Journal of Economics*, which is also quoted in the book.

A square matrix of technical coefficients $A = \{a_{ij}\}$ is defined, and its determinant [illegible] equals zero, due to the homogeneity of the system and also because the technical and consumption coefficients are not independent: the relation of the services supplied by the households with their remuneration (thus the consumption coefficients) is determined by the productivity, reflected by the production coefficients. By virtue of the latter, any price can be expressed in terms of the cofactors of the elements of matrix *A*:

$$P_{i1} = [illegible]$$

P_{i1} is the price of good *i* expressed in terms of commodity *1* and [illegible] and [illegible] are respectively, the cofactors of elements a_{1i} and A_1 in determinant [illegible]

In a similar fashion to prices, equilibrium quantities can also be estimated in terms of the technical and investment (or saving) coefficients as independent variables. The savings ratio is obtained dividing the total revenue between the total revenue and the total purchases of the consolidated households' account (later this ratio will need to adjust as to make the determinant of the related matrix zero). The savings and investment coefficients adjusts when there are discrepancies between the consolidated households' revenues and expenditures

The system is also homogeneous of degree zero and can be solved for the relative values of quantities X_1, X_2, ... , X_n. The system assumes that the production and consumption functions are homogeneous and the amounts of every commodity are variable and undetermined; that means that production and consumption functions are linear and the economies of scale are constant, while quantities are relative only. Leontief refers to Cassel's (Cassel, 1918) idea about the existence of lump initial amounts of basic inputs (e.g. natural resources), as considerably shaken, although this author is not mentioned directly. That idea would introduce non-homogeneities to the

production functions, while it is not very significant. Indeed Leontief states that the actual economic system would be a large set of homogeneous linear equations and a single non homogeneous member (p. 49).
The proposed system is consistent provided that the determinant (D) of matrix *A* (of technical coefficients) equals zero. That is, at least one coefficient is not independent and then the savings ratio can adjust in such a way as to make the system consistent. The savings ratio can also be read as an investment coefficient, which will affect the production conditions in each industry; that in turn is determined by two factors, the specific investment and savings conditions in each industry and the phenomena that affect investment to all the set of industries. When the investment coefficients are all equal to 1, D = [illegible] = 0.
In sum, relative prices and relative quantities are functions of the set of coefficients and changes in the latter change prices and quantities. The effects of such changes of the independent variables on the dependent ones can be measured through the corresponding partial derivatives; nevertheless there are over one hundred parameters in the empirical cases presented in this book. Therefore Leontief advises to restrict the research to a few exemplary cases, setting also the initial conditions. Leontief suggests giving particular attention to the case when there is a change in the productivity of a given commodity in all its uses as a cost factor of other industries, which would change all the proportions in the system, including relative prices and quantities, proportionately to the differences between the former and the latter productivities.

- The title of Part III is "Data and variables in the American Economic System, 1919-1929" and it is devoted to present and discuss the numerical solutions of the system presented in Part II for the 1919 and 1929 US economy. The IO tables are printed and folded in a pocket on the back cover; they are disaggregated into 44 accounts, including households, international trade and the undistributed account. Nevertheless, for "technical reasons", which must be related to the difficulties of computing such large matrices, the calculations are made on ten industries arrays.

Using the household services as *numéraire* and defining a physical measure of all commodities in terms of the physical amounts that can be bought with one currency unit, the system can be understood as defined in relative physical terms. Since the Matrix is known for two years, all the coefficients can be computed, including the determinants D and [illegible]
Nevertheless, the amount of arithmetical operations, Leontief warns, is very high, which is also time consuming. Fortunately Leontief had access to the Calculator developed by Professor John B. Wilbur of the Massachusetts Institute of Technology; such machine offers (offered?) the possibility of solving systems of up to nine linear simultaneous equations with real coefficients. The machine and its way of operation is described in "The Mechanical Solutions of Simultaneous Equations" by John B. Wilbur, published in the *Journal of the Franklin Institute*, Vol. 22 No. 6 (December, 1936) pp. 715-24 and this article deserves the only reference in the book. Leontief explains that the early computer calculates the ratio of two determinants interrelated in a certain

way.
Further in order to analyse the evolution of the sectors in the US economy between 1919 and 1929, as well as to the test the model he proposes, there is a lengthy presentation of the results of the reactions of prices, quantities and coefficients to changes in the productivity, the investment and saving coefficients. One recurrent feature is that of the influence that changes in one industry can represent on the relative prices or quantities of other sectors, by virtue of the interdependence of all the sectors. It is interesting to note that despite this early use of the IO model concerning structural changes, the most extended criticism against it has been its static character. Of course Leontief stated the latter, which was reinforced by the assumption of the rigid discontinuous isoquants and the corresponding production function.
Also the analysis is made in terms of the households "industry" because so far, *The Structure of the American Economy* presents a "closed" model; nevertheless, implicitly Leontief assumes that the production system's purpose is to satisfy the aggregated demand function, in accordance with Cassel's (1918) statement. It is worth mentioning here a general conclusion that Leontief reaches with his analysis at this point, namely that the economic system is basically stable. In fact, in a footnote on page 79, it can be read that "The prevailing belief in the inherent instability of economic phenomena seems in large part to be the result of a singular optical –or rather psychological- illusion. As a moving object, however small, is likely to distract the attention of the casual observer from the static features of a vast landscape, so the changing elements of the economic scene attract very often the attention of investigators to the exclusion of other, less dynamic elements of the picture..."
The first edition of *The Structure of the* American *Economy* ends discussing the some special problems, basically the problem of industrial classification, which is in the end arbitrary, but determines in some extent the results the researcher gets in most empirical exercises in the field of applied economics. In fact, Leontief presents alternative classifications and so different results. Finally there are a few considerations regarding the effects of changing the consumption patterns on the coefficients of the system, as well as on prices and quantities, although not all those results were actually computed nor discussed.
The first edition of *The Structure of the* American *Economy* has the structure of a long article, not having an introductory or a conclusive chapter. The book actually reaches its purpose, namely, to present a *Tableau Économique* for the US economy in 1919 and 1929. Not only that, but also the author performs a number of calculations that allow him to assess the

- In February 1944 in the *Quarterly Journal of Economics* (Vol. 58 No. 2) Leontief published "Output, Employment, Consumption and Investment", which would later be reprinted as section A of "Part IV, Applications of Input-Output Technique to the American Economic System in 1939" of the second edition of *The American Structure of the American Economy 1919-1939.* This paper deals with the issue of reconverting the war economy to a peace one and the possible effects of such transition on the output and employment levels if the

government ceased to purchase war materials and no replacement would be found.

Thence, the article describes a method of estimating the quantitative relationships between demand for the produce of each branch, industry or sector and the economy total output and employment. The first assumption for the analysis is the one that would become fundamental in the IO analysis, namely that the productive sector works in order to satisfy consumption. Secondly, technology is also central to explain the functioning of the economy; hence Leontief says that "Given the annual bill of goods which is to be made available for consumption (and for new investment), the total outputs of various industries requisite for its actual production depends primarily upon the technical structure of all the many branches (…) which directly or indirectly contribute to the output of the various commodities included in this final bill of goods… (p. 139).

As it is well known nowadays, employment can also be linked to the final demand level via output level, given the *technical structure* (italics in the original) of all the industries. Equilibrium employment is determined in a uniquely way, distributed also uniquely among all the sectors and branches, associated with a given bill of goods. The author concludes that employment policies can only be successful by affecting these relationships.

Hence, economic policies should modify the volume and direction of income and investment streams. The magnitude of the final effects of such policies, depend on the character of the structural relationships, which cannot be affected by the government actions. Leontief is thus sceptical in regards to the effectiveness of policy measures aimed at modifying equilibrium results. Nevertheless, accurate knowledge of the structural relationships is indispensable for appraising the probable results of intervention policies.

The method to compute the total (gross) output and employment for all the industries and a given bill of goods assumes there is one (and only one) combination of outputs which makes the output of each industry large enough to satisfy its final (consumption plus investment) and intermediate demands (in modern language). The employment of labour is also uniquely determined.

Leontief modifies the system of equations (I) presented above by rewriting it as:

(I)

$$+X_1 - x_{21} - x_{31} - \dots - x_{i1} - \dots - x_{m1} = x_{n1}$$
$$-x_{12} + X_2 - x_{32} - \dots - x_{i2} - \dots - x_{m2} = x_{n2}$$
$$-x_{13} - x_{23} + X_3 - \dots - x_{i1} - \dots - x_{m3} = x_{n3}$$
$$\dots$$
$$-x_{1n} - x_{2n} - x_{3n} - \dots - x_{in} - \dots + X_m = x_{nm}$$

further, he defines the technical inputs coefficients as usual:

$$a_{ik} = x_{ik}/X_i$$

and rewrites system (I) as:

(III)

$$X_1 - a_{21}X_2 - a_{31}X_3 - \dots - a_{m1}X_m = x_{n1}$$

$$-a_{12}X_1 + X_2 - a_{32}X_3 - \ldots - a_{m2}X_m = x_{n2}$$
$$-a_{13}X_1 + a_{23}X_2 + X_3 - \ldots - a_{m3}X_m = x_{n3}$$
$$\ldots$$
$$-a_{1m}X_1 - a_{2m}X_2 - a_{3m}X_3 - \ldots + X_m = x_{nm}$$

The last column of this system stands for the bill of goods, which is given for the purpose of solving it, as it is the set of technical coefficients. The unknowns are the industry outputs $X_1, X_2, \ldots, X_m$. The solution of (III) is thus written as:

(IV)

$$X_1 = A_{11}x_{n1} + A_{12}x_{n2} + \ldots + a_{1m}x_{nm}$$
$$X_2 = A_{21}x_{n1} + A_{22}x_{n2} + \ldots + A_{2m}x_{nm}$$
$$X_3 = A_{31}x_{n1} + A_{32}x_{n2} + \ldots + A_{3m}x_{nm}$$
$$\ldots$$
$$X_m = A_{m1}x_{n1} + A_{m2}x_{n2} + \ldots + A_{mm}x_{nm}$$

Coefficient A_{12} are described as showing "… by how much the output of industry *1* would increase (or decrease) if the amount x_{n2} of commodity *2* which is entered into the bill of goods is raised (or reduced) by one unit (while the remaining parts of the bill of goods remain unchanged)." (p. 145). Such definition would later be linked to that of the multipliers and each A_{ik} depend upon the set of technical coefficients in system (III). A_{ik} is defined as:

$$A_{ik} = D_{ik}/D$$

while D is the determinant

and D_{ik} is the corresponding minor.

Before continuing revisisng the contents of the book it is interesting to note that system (III) is clearly an open system, for which final demand is no longer determined within the system. Therefore there is no longer a justification for the existence of stable or given consumption coefficients, although some attention is given to defining the concept. On the other hand, despite the fact that Leontief does not present the more common solution presented in present day text books, together with the well known discussion (e.g. Miller and Blair, 2009), he presents the IO model as it known nowadays, quite different from the 1941 version (see above), although the multipliers are defined differently.

Employment for is computed from the employment coefficients and the output equations. For industry I

$$(V)\ x_{1n} = a_{1n}A_{11}x_{n1} + a_{1n}A_{12}x_{n2} + \ldots + a_{1n}A_{1m}x_{nm}$$

and the total employment for the economy equals the summation over the set of industries:

$$(VI)\ x_{1n} + x_{2n} + \ldots + x_{mn}$$

As a conclusion of these equation is that employment policies should deal with output levels.
In this chapter Leontief presents the 1939 IO table for the US, both in money terms and the technical coefficients matrix. The first table, he comments, can be seen as a table expressing physical demands for inputs, then, there is the technical coefficients table for ten industries plus the households services: employment that enters as a row. The solution of system (III) for ten simultaneous equations was reached using a Marchand computing machine, which could take some five hours, according to a footnote on page 150.
In order to explore the effects of technical changes on the empirical results of the model, Leontief proposes to solve it using the 1939 coefficients table and the 1929 output data. Such an exercise, the author says can also be used to predict future outputs on the bases of expected or known bill of goods. Discrepancies between the observed and the calculated results can be explained by the differences between both technical tables, which can be attributed to technical change or to differences in the classification criteria.
Finally the chapter presents the results for the employment model that had previously been discussed. Those results are naturally linked to final demand levels, which determine output in every industry. It has been customary to link the employment model to the keynesian perspective of effective demand. Nevertheless, this can also be linked to the Cassel definition of the economic system as devoted to satisfy the consumers' necessities.

- In *The Quarterly Journal of Economics* (Vol. 60 No. 2, pp. 171-93), February, 1946, the article "Exports, Imports, Domestic Output and Employment" by W. Leontief was published, which would later be included in section B of "Part IV, Applications of Input-Output Technique to the American Economic System in 1939" of the second edition of *The American Structure of the American Economy*. There would be an errata to this article published in the same journal, Vol. 60, No. 3 (May, 1946), p. 469; in the book this errata is naturally included. This paper explores first the connections between foreign trade and the output and employment levels. The tools that Leontief suggests for that task are the same that were analysed in Section A of part IV of this book. In effect, the external sector is treated as one industry, with imports representing outputs and exports representing inputs. Imports must be classified between competitive and non competitive; further the former can be assigned to the sectors producing similar goods which will be consumed by the various industries regardless of the origin of the inputs (imported or domestically produced). Hence, the imports row shows a combination of imports absorbed by the consuming industries. Similarly, trade margins can be charged together with the price of the consumed commodity.

Exports are not treated as a part of final demand mainly because they are instead, determined by the level of output produced in the country. Exports must be as large as necessary to finance the imports required by the output size, which is also determined by the terms of trade, given the technical conditions of production within the country,

as well as the structure of the industry called "Foreign Trade". Therefore, only the export surplus can be treated as an independent variable. Such a variable will behave and affect the rest of the variables, as does any other source of final demand. As a conclusion, any variation of the final demand will affect the economic system in similar fashion; thus they should be treated mathematically in the same way to calculate the effects on the output levels.
The paper presents the formal treatment of the analysis and then there is a detailed discussion of the results for the US economy in 1939. The effects on imports is also taken into account and the effects of changing final demand and its components are also calculated. Such changes allow Leontief to characterise the US economy and its various industries in 1939.

7. The third paper included in the second edition of *The Structure of the American Economy* is "Wages, Profits and Prices" which first appeared in the *Quarterly Journal of Economics,* Vol. 61, No. 1 (November, 1946), pp. 26-39. The purpose of this paper is to "… present the measures of certain fundamental interrelationships between the wage rates paid, profits derived and the prices received by all the various branches of the American economy during the last normal pre-war year, 1939." (p. 188).
The relationship between costs and prices in each industry is mediated by the technical conditions of production. The quantitative relationship between the price P_i of good *i*, produced by industry *i*, i = 1, 2, …, *i*, … *m*, and the prices of the inputs employed in its production process is as follows:

$$a_{i1}P_1 + a_{i2}P_2 + \ldots + a_{im}P_m + R = P_i$$

R_i is the value added per unit of output in industry *i*. Leontief takes a cost-price theory from an accountant practice (p. 189). This is also Cassel's proposal for explaining prices of production of good *i* (Cassel, 1918). Then it is possible to postulate a system of *m* equations for the *m* prices in the system.
If technical coefficients are known, there are 2m unknown variables, the *m* prices and the *m* R_is. If the value added for each industry is known, the system can be solved for the prices; otherwise, if prices are known, value added (and its distribution between wages and profits) in each industry can be found within the equation system. In order to determine the distribution, it is necessary to know also the technical relationships between factors as well as the factoral technical coefficients.
The solution of the former system for the *m* prices, in terms of the vector of value added would be:

$$P_1 = A_{11}R_1 + A_{21}R_2 + \ldots + A_{m1}R_m$$
$$P_2 = A_{12}R_1 + A_{22}R_2 + \ldots + A_{m2}R_m$$
$$\ldots$$
$$P_m = A_{1m}R_1 + A_{2m}R_2 + \ldots + A_{mm}R_m$$

Where each A_{ki} depend upon the matrix of technical coefficients (*vid. supra*.); A_{ki} shows the relationship between P_i and R_k, that is A_{ki} is a price multiplier on the value added. If R_i is splitted between wages and profits, it is possible to have separate results on the influence of wages and profits variations on the prices in each industry, provided that

the labour and capital coefficients are available.
The chapter goes on explaining a few simulations of rising (or reducing) separately wages and profits in 10% for all industries and their effects on prices in the US economy in 1939. The second set of simulations, consider increasing (or decreasing) wages and profits in each industry at the time. These simulations assume fixed technical coefficients. As expected, despite the uniform treatment of all the industries, results for each one are different, which can be explained by the differences in the relationships between each industry and the rest of the economic system.

It is interesting that Leontief explains the meaning of keeping technical coefficients fixed and mentions the validity of Pareto's criticism on the Walrasian fixed technical coefficients. Although there is no actual citation about this controversy, it might be useful to remind that H. Neisser published a paper "A Note on Pareto's Theory of Production" in *Econometrica,* Vol. 8, No. 3 (July, 1940), pp. 253-262, who in turn quotes H. Schultz "Marginal Productivity and the General Pricing Process" in the *Journal of Political Economy*, Vol. 37, No. 5 (October, 1929), pp. 505-551. The former article is quite contemporary to Leontief's publication and might be a source of that remark. All that is also a pretext to discuss the quality of the available statistical data as well as the frequency that the organisms in charge can take to update it. Statistical data can be a source if error in any calculation and prediction. This is a problem that despite the modernisation of the statistical offices all over the World, might keep its relevance.

8. In *The American Economic Review* Vol. 39, No. 3, Papers and Proceedings of the Sixty-first Annual Meeting of the American Economic Association (May, 1949), pp. 211-225 , Leontief published the article "Recent Developments in the Study of Interindustrial Relations" which would be included as Section D (and last) of Part IV of *The Structure of the American Economy 1919-1939*. This paper was first presented in the Annual Meeting of the American Economic Association, held in Cleveland, Ohio, USA, December 27-30, 1948. It was commented by Solomon Fabricant, Irwin Friend, Walter Jacobs, Marvin Hoffenberg, Tjalling C. Koopmans, Raymond W. Goldsmith and Oskar Morgenstern.
The paper is a public presentation of the IO model thus, it somehow summarises the discussion held along the book, although it contains new insights and discussion. Leontief commences remarking the importance of seeing the economy as a system of interrelated industries and the connection of that view with the construction of the IO table. The theoretical bases of the idea of an economic system lay on the general equilibrium theory, mainly because the input-output model aims at studying the relationships between industries, i.e., the economy as a system, not a single industry, let alone a firm.
It is stated that the practical model is an open one, which means that in order to solve it, it is necessary to set the values of some variables arbitrarily. The general logic of that procedure is alike to different Keynesian multiplier models. The consumers and governmental purchases (final demand) can then be fixed and through a system of simultaneous linear equations, the rest of the variables can be computed, keeping the set of coefficients fixed. Labour, capital and other services produced by the final

demand consumers must be treated as primary (non produced) inputs. By that device, however, many essential connections are lost, such as that between the employment and the income levels; nevertheless, the model will always maintain a number of proportions that allow solving it. So the open IO model defines a unique relationship between the set of prices, the wage rates, the set of profit rates. Given two of them, the third can be computed.

Leontief then discusses on the need to aggregate various activities and how that might be undesirable when analysing some features. Next he explains that techniques of statistical inference, correlations and multivariate regressions are linked to Keynesians models and studies. Correlation and aggregation go together since those methods work only with highly aggregated quantities; they however obscure the structural relationships. On the contrary, empirical general equilibrium would only progress if it were possible to rely upon methods and statistics that consider disaggregated variables.

Finally the book closes with some considerations regarding the hypothesis of constant technical coefficients, which by the time seems to have been already a constant criticism against the IO framework. As usual, Leontief considers various arguments, for and against and even makes a few numerical tests to measure the validity of that thesis. Both following a logic and theoretical discourse and by testing numerical results, Leontief shows that those criticisms lack of any bases. It is reasonable to expect that technical coefficients change slowly, as it has been shown later by various other authors (vg. Carter, 1970).

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